To: Ann Anderson, Union College Professor
From: Roderick Landreth
Date: 18th of February, 2019
Subject: **Heat Transfer Analysis of Two Potatoes Cooking**

Analyses of cooking potatoes via baking and boiling produced a ranges of times for the core to reach 90^oC. Potatoes were then baked and boiled with type K thermocouples monitoring core temperatures to confirm the analytical calculations. The potato boiling in 100^oC water reached 90^oC faster than the oven-baked potato in 204.4^oC air, as seen in Figure 1.

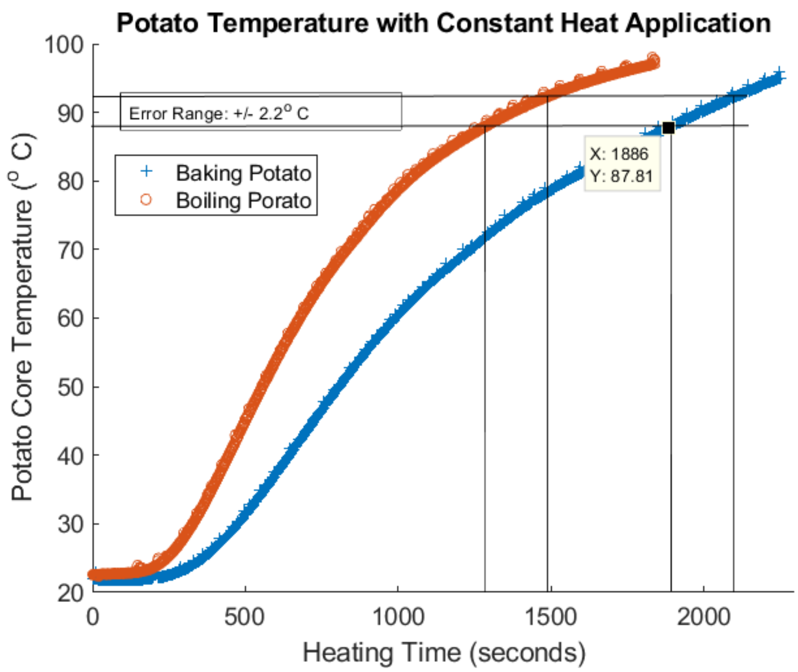

Figure 1: Temperature Plot of Potato Cores with type K Thermocouples at 0.5Hz, in this temperature range with an error of $\pm 2.2^oC$ [1]. Boiling time: 1378 $\pm$ 94(s). Baking time: 1988 $\pm$ 101 (s)

Data Comparison and Summary

The analytical evaluation of the time it takes the baked potato core to reach 90^oC produced 1124 and 4050 seconds, using the lowest and highest possible Biot numbers. The Biot number (equation 1 Appendix C) characterizes the ratio of conductive thermal resistance to the convective resistance. The lower Biot number shows a larger heating time, indicating a relatively larger convective thermal resistance. Figure 1 displays 1988 $\pm$ 101 seconds for the experimentally baked potato, easily encompassed within the analytically calculated range of times. Similarly, the boiled potato was calculated to have an average cooking time of 1,122 seconds, compared to the experimental time of 1378 $\pm$ 94 seconds is 4.7% different (equation 2 Appendix C). This is within the range of analytically calculated times, considering the variability of the Biot number with environmental conditions. (Experimental Procedure and Pictures in Appendix A)

Results Analysis

The Biot number varies largely with the values from Appendix A for the potato's thermal diffusivity, conductivity, and the convection coefficient h. Both experiments use a similar potato, so the thermal diffusivity and conductivity will be similar. This experiment mainly displays the effect of using fluids with different convection coefficients to transfer heat to a solid. Newton's Law of Cooling (equation 3 Appendix C) shows that a primarily convective boundary's energy transfer relies on the temperature difference and the convection coefficient. The forced convection coefficient range for liquid is orders of magnitude larger than that of free convection in gas, so the smaller temperature difference in the boiling water still transfers more heat. An interesting follow up experiment would be to bake a potato in a convection oven with forced flow, or in still hot water.

Analytical Solution Procedure

The range of the baked potato's physical properties and environmental conditions as provided in appendix B produce a range of Biot numbers. The lumped capacitance method is accurate for the system within the lower Biot number range, and the approximate analytical solution, the Mid-plane formula (equation 4 Appendix C) is feasible for the higher Biot range. The lumped capacitance method produced 4050 seconds, compared to the mid-plane formula's 1122 seconds. This large variation may reflect that the combination of values assumed for lumped capacitance may not be realistic. The lowest Biot number refers to a highly conductive potato in still, hot air while the larger Biot number corresponds to a less conductive potato in slightly flowing air, closer to realistic conditions. Calculations of these solutions are in appendix C. The approximate analytical solution for a boiling potato (the midplane formula) assumed the average Biot number from the given property ranges, and required the Fourier's number to be greater than 0.2. After the calculation, the Fourier's number was confirmed to be within an acceptable range.

Error Analysis

These large ranges of values in table 1 (Appendix B) take into consideration the variability of the fluid conditions in the boiling water and the hot air, though do not encapsulate sources of error during testing. Fluctuating temperatures and conductive boundaries altered heat transfer rate in the experimental tests. Thermocouple temperature error added to uncertainty in cooking time, as well. Analytical calculations assumed an ambient temperature of 23^oC, a baking temperatures of 204.4^oC, and a boiling temperature of 100^oC. In addition, the the potatoes were assumed to be spheres, properties to be constant, relatively small amounts of radiation and conduction, and uniform conditions. The temperature of boiling water is fairly consistent, but the oven runs in heating cycles. Appendix B shows a plot of oven air temperature vs. time in a 10 minute period. The average temperature of a heat cycle is 220.43^oC, much higher than the set temperature.

If any assistance is needed with replicating experimental conditions, I would be glad to help.

Appendix A

Experimental Procedure:

1. Keep potatoes in the same place overnight to ensure same, room temp interior
2. Measure room temperature
3. Place a large amount of water in a pot and heat to rolling boil
4. Puncture potato with thermocouple, inserting it up to its radius in. Place some hot glue on the outside to stop water from entering and effecting temperature readings.
5. insert potato in water, keeping it centered in pot. If water runs low, slowly add fresh water while increasing stove temperature, away from potato.
6. Record till center is $90^o C$, take time measurement and remove from water
7. Preheat oven to $400^o F$
8. Wait till preheated, make a temperature plot of the oven for 10 minutes
9. Insert potato into the top rack of the oven with thermocouple inserted
10. Record till center is $90^o C$, take time measurement and remove from oven
11. Make potatoes into perogie filling, optimally.

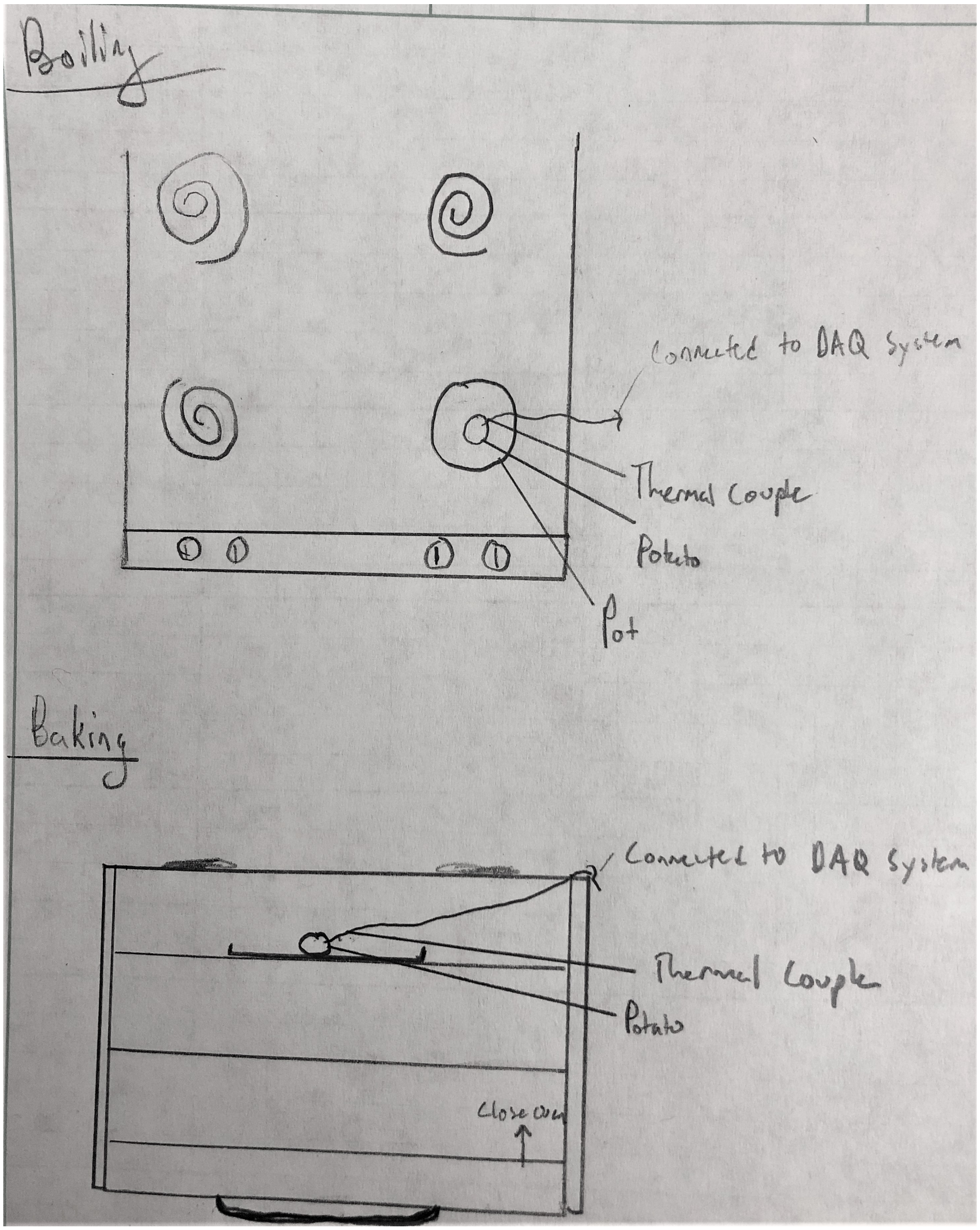

Figure 2: Experimental Setup Plan

Figure 3: Data Acquisition Setup for Baked Potato on a baking sheet. Contact with the baking sheet will have altered heat transfer.

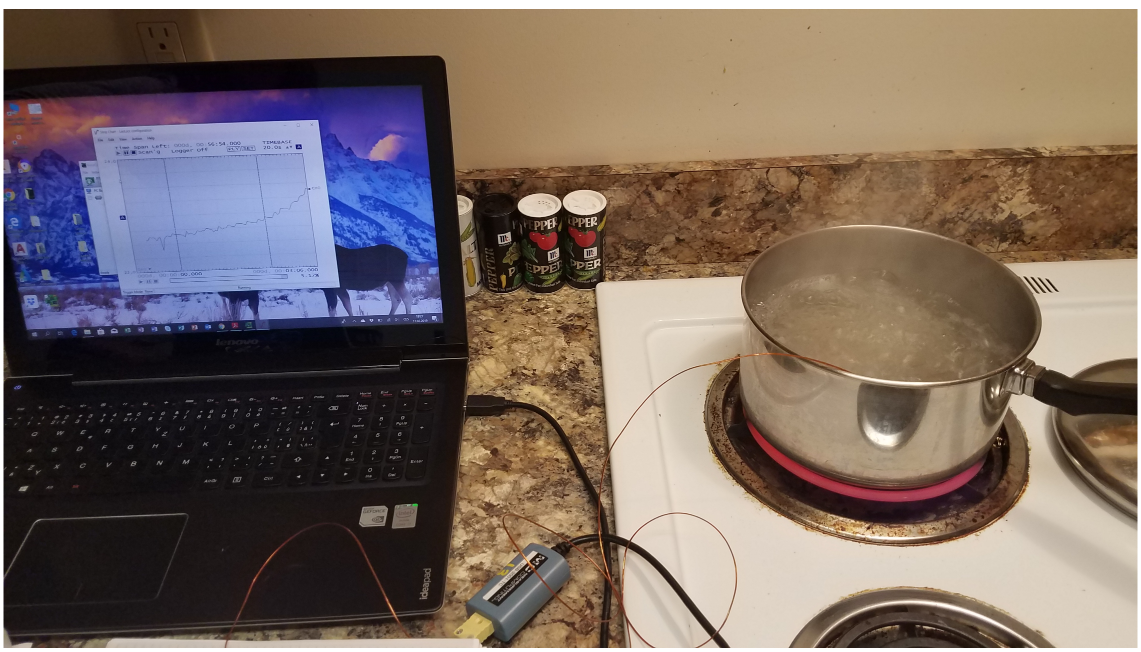

Figure 4: Data Acquisition setup for Boiled Potato, example of bubbles pushing the potato to the side and in contact with the metal pot.

Appendix B

Property	Boiling	Baking
thermal diffusivity, α_{potato}	1.08×10^{-7}- 2.7×10^{-7}	1.08×10^{-7}-2.7×10^{-7}
thermal conductivity, k_{potato}	0.4-1.0 W/mK	0.4-1.0 W/mK
ambient temp, T_∞	100 °C	400 °F
heat transfer coefficient, h	1000-6000 W/m^{2}K	2-15 W/m^{2}K

Table 1: Properties of Potatoes and the Environments Used to Cook Them [2]; Biot Numbers (equation 1 Appendix B) range from 0.055 to 1.023 for the baked potato, and from 27.7 to 415.5 for a boiled one.

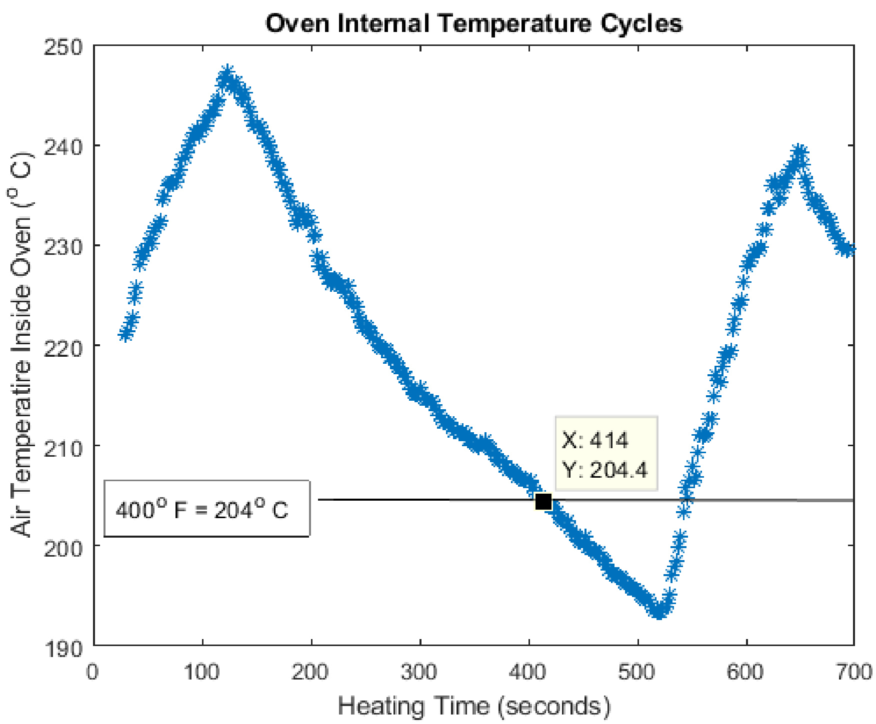

Figure 5: Non-Constant Oven Temperature, One Cycle. Average Temperature = 220.43^oC

Appendix C

Equations:

$$Biot\ = \frac{hL_c}{k} \tag{1}$$

$$\%Diff = \frac{A-B}{0.5(A+B)} * 100 \tag{2}$$

$$\dot{q}" = h(T_s - T_\infty) \tag{3}$$

$$\frac{T-T_\infty}{T_i-T_\infty} = C_1 e^{-\zeta^2\ Fo\ Cos(\zeta_1, x^*)} \tag{4}$$

Values for ζ and C_1 from table 5.1 [3]

Work Shown:

Boiling

$r = 2.77\ cm = 0.0277\ m$

$\alpha_{avg} = 1.89 \times 10^{-7}\ \frac{m^2}{s}$

$k_{avg} = 0.7\ \frac{W}{mK}$

$h_{avg} = 3500\ \frac{W}{m^2K}$

$Bi = \frac{h\, r_0}{k} = \frac{3500(0.0277)}{0.7} = 138.5$ → Table 5.1 - use values for ∞

$\Rightarrow\ \xi_1 = 3.1415$

$C_1 = 2$

$\theta^* = C_1 \exp(-\xi_1^2 F_0)$

$\frac{T - T_\infty}{T_i - T_\infty} = C_1 \exp(-\xi_1^2 F_0)$

$\frac{90-100}{23-100} = 2 \exp(-3.1415^2 F_0)$

$F_0 = 0.277 > 0.2$ ✓ $\Rightarrow$ Approximate Solution is valid

$F_0 = \frac{\alpha t}{r_0^2}$

$t_{avg} = \frac{F_0\, r_0^2}{\alpha} = \frac{0.277(0.0277^2)}{1.89 \times 10^{-7}} = \boxed{18.75\ min}$

Figure 6: Approximate Analytical Solution, Mid Plane Formula for a 23^oC Potato in 100^oC water

Baked potato Approximate Analytical:

w/ $r = 2.73\,cm$, $T = 90°C$
$\alpha = 1.89 \cdot 10^{-7}$, $T_i = 23°C$
$k = 0.4\ W/mK$, $T_\infty = 204°C$
$h = 15\ W/m^2K$

(properties chosen from a range to maximize Biot#)
-Assuming $Fo > 0.2$

- $\text{Biot}\ \# = \frac{h\, r_0}{k} = \frac{15 \cdot 0.0273}{0.4} = \underline{1.024}$

$1.024 > 0.1$, $\underline{\text{can't}}$ use Lumped:

- $\theta^* = C_1 e^{-\zeta_1^2 \cdot Fo}$

$\zeta_1 = 1.5708$ $\quad C_1 = 1.2732$
(table 5.1)

$$\rightarrow \frac{\ln\left(\frac{\theta^*}{C_1}\right) \Delta x^2}{-\alpha \zeta_1^2} = \frac{\ln\left(\frac{0.6298}{1.2732}\right) \cdot 0.0273\,m^2}{-1.89 \cdot 10^{-7} \cdot 1.5708^2} = \underline{1124.8\,s} = 18 \text{ minutes } 44 \text{ seconds}$$

$$\theta^* = \frac{T - T_\infty}{T_i - T_\infty} = \frac{90°C - 204°C}{23°C - 204°C} = 0.6298$$

- Recalculating using α_{min} & α_{max}, the minimum and maximum times are 1968s (32 min) and 787s (13 min)

- $Fo = \frac{\alpha t}{L_c^2} = \frac{1.89 \cdot 10^{-7} \cdot 1124.8\,s}{(r_0)^2} = 0.285$

$Fo > 0.2$ ✓

18 minutes 44 seconds

Figure 7: Approximate Analytical Solution, Mid Plane Formula for a 23^oC Potato in a 204.4^oC oven

Baking - Lumped Capacitance Method

$r_o = 0.0273\ m$

$B_i = 0.05 < 0.1$ ✓

$$\frac{T_\infty - T_f}{T_\infty - T_i} = e^{-\frac{B_i \alpha}{L_c^2} t}$$

$$L_c = \frac{r_o}{3} = \frac{0.0273}{3} = 9.1\ (10^{-3})\ m$$

$$\frac{204.4 - 90}{204.4 - 23} = e^{\frac{-0.05\ (1.89)(10^{-7})}{(9.1\ (10^{-3}))^2} t}$$

$$\underline{t = 67.5\ min}$$

Figure 8: Lumped Capacitance Solution for 23^oC Potato in a 204.4^oC oven

References

[1] *USB-2001-TC User Guide*, Measurement Computing Corporation, www.mccdaq.com/PDFs/manuals/USB-2001-TC.pdf, Accessed 19 Feb. 2019

[2] Ann Anderson, *Lab 3: Cooking Potatoes – Boil or Bake?*, 2019, Lab handout.

[3] Bergman, L., Dewitt, D., Incroperam, F., Lavine, A., *Introduction to Heat Transfer,* 6th Edition, 2011