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MER-322: Dynamics of Physical Systems
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Torsion Plant System Modeling

Introduction

This exercise compared the numerical model of angular velocity of a torsion plant run in the two disk configuration to the physical system behavior. The system comprised one weighted disk at either end of a vertical steel rod, with a motor applying torque to the bottom disk. The system parameters were determined through experimental testing, including torsional spring constant of the steel rod, the moment of inertia of each disk, and the damping constant on each disk. The numerical model's simulation was contrasted to velocity readings from the system to determine the model's accuracy.

Methods

A motor applied torque on the bottom disk as the system's input and the desired output was the angular velocity of the top plate. Modeling the four state system with Simulink started with deriving equations of motion from free body diagrams. The state equations were functions of the four state variables (Θ_1, $\dot{\Theta}_1$, Θ_2, $\dot{\Theta}_2$), constants of damping (B_1 &B_2) and a torsional spring (K_1). Simulink modeled the system discretely with state propagation, specifically the Runge-Kutta method with a timestep of $1 * 10^{-6}$ seconds. The input for the system model was a step function, stepping from 0V to 0.5V for 0.1 seconds before stepping back to zero every 3 seconds. This repeated for 6 periods.

This numerical model was then compared to the physical system reacting to the same torque input. The data generated by the sensor was angular displacement over time, so the data was processed to display the angular velocity, calculated as the change in position per change in time.

Results

Figure 1 displays the numerical model's correlation to the physical system dynamics. The system's input is indistinguishable from the model's input, and both displayed a similar pattern of velocity oscillations. Even so, there are differences between the system model and the physical system behavior. The numerical model's velocity curve has maxima lower than the physical system, with a small phase shift. In addition, the region after t = 5.5s in Figure 1 displays the model's velocity magnitudes decaying slightly faster than the observed system, the physical system dropping to zero where the model still oscillates.

Attachment A depicts two alterations of the system input that causes more complex system dynamics, and shows how the numerical model changes with the same input. Reducing the time in between voltage pulses prevents the velocity from reaching zero, in Figure 2 causing constructive interference that slowly increases the maximum velocity of the disk. Changing the pulse frequency to act on the disk as the velocity is decreasing and negative causes destructive interference instead. The peak velocity magnitudes decrease each period, as seen in Figure 3. Attachment A plots also match Figure 1 in that the physical system and the model contain a small phase shift, and the maximum velocity values are higher in the physical system.

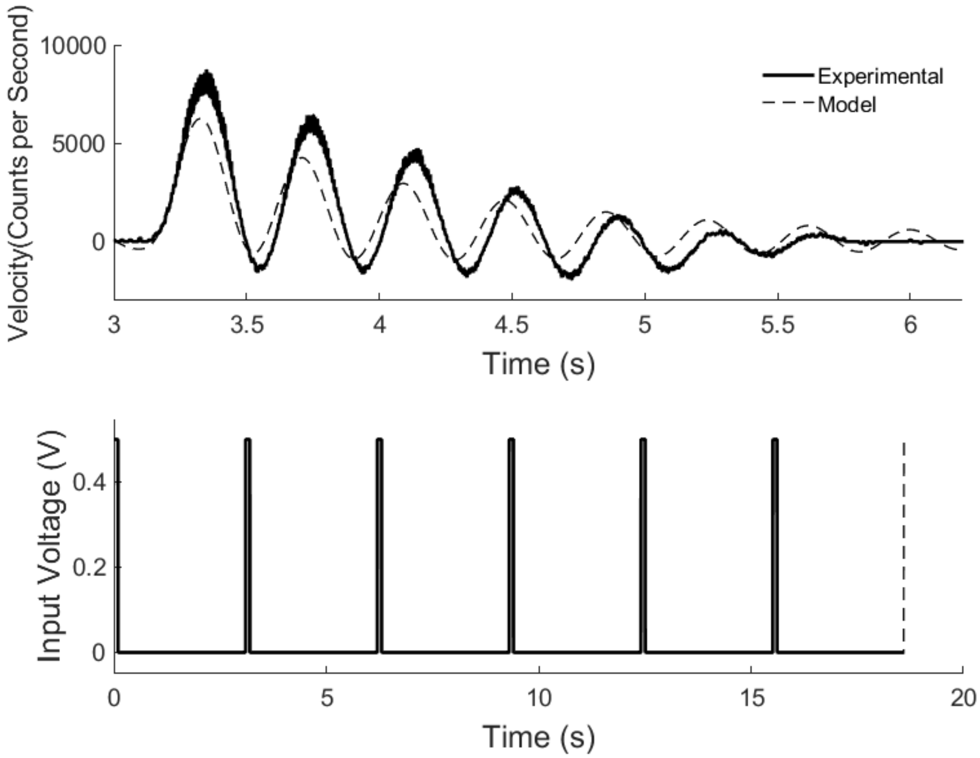

Figure 1: A Comparison of the Numerical Model's Angular Velocity Output to the System's Realistic Behavior. System parameters include $k = 1.3836\frac{Nm}{rad}$, $B_1 = 0.027\frac{Nms}{rad}$, $B_3 = 8.2 * 10^{-4}\frac{Nms}{rad}$, $J_1 = 0.0103kgm^2$, and $J_3 = 0.0099kgm^2$. Bottom: The motor received input in voltage, while the system received torque equal to 0.6 times the voltage. This graph confirms similar system inputs. Top: one count is 1/16000th of a full rotation. The derivative of the position vs time was not smooth or continuous, so a moving average filter with a smoothing factor of 1/6 reduced the noise in the angular velocity plot for easier comparison.

Discussion

The state equations for the system model assume zero static friction and quadratic air resistance. These damping elements would cause the model's velocity to decay faster, matching the physical system's response more closely. The lower magnitude of the numerical model's velocity maxima are due in part by this oscillation that remains before each pulse. If the velocity is in the negative direction during a pulse, destructive interference would decrease the magnitude of velocity resulting from that pulse. In addition, the step function input jumps immediately from zero to 0.5V, where the function generator within the physical mechanism upon closer inspection can not create an instantaneous jump, but a steep slope up to 0.5V. Although the inputs are seemingly indistinguishable, this would slightly change the power input into the system.

By the property of superposition, Figures 2 and 3 can be recreated by overlapping the effects of a single pulse on top of another, with a phase shift of the dwell time. It makes sense that the same differences in between the model and system in Figure 1 would carry through to plots of each manipulation of the input, because the only thing being altered is the space in between pulses, not the model.

Attachment A

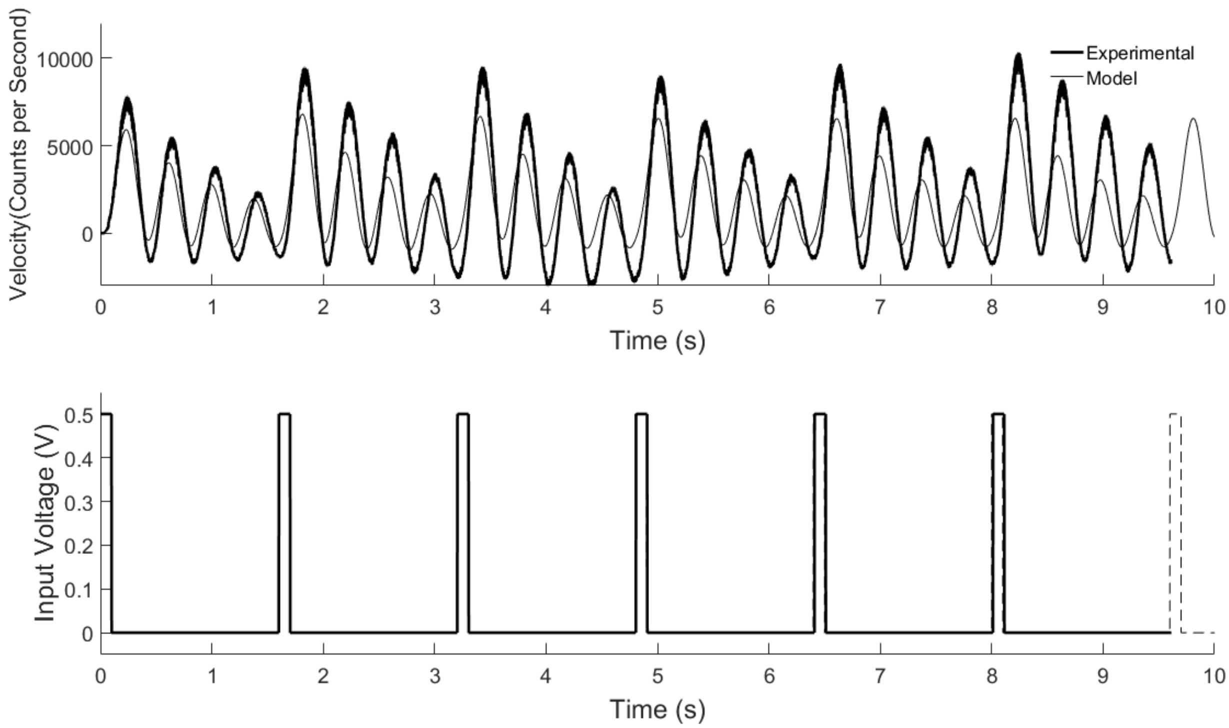

Figure 2: A Comparison of the Angular Velocity Output from Between the System and the Numerical Model with Altered Input. Bottom: The motor received input in voltage with a dwell at zero for 1.5 seconds, and the pulse at 0.5V for 0.1s. The system received torque equal to 0.6 times the voltage. This graph confirms similar inputs between the model and physical system. Top: The output response shows positive feedback resulting from the torque application causing constructive interference. The maximum value of the velocity of the first period at $t = 0.24s$ is 8417counts/s, vs. at $t = 8.24s$, v=11000counts/s. One count is 1/16000th of a full rotation. A moving average filter with a smoothing factor of 1/8 reduced the noise in the angular velocity plot for easier comparison.

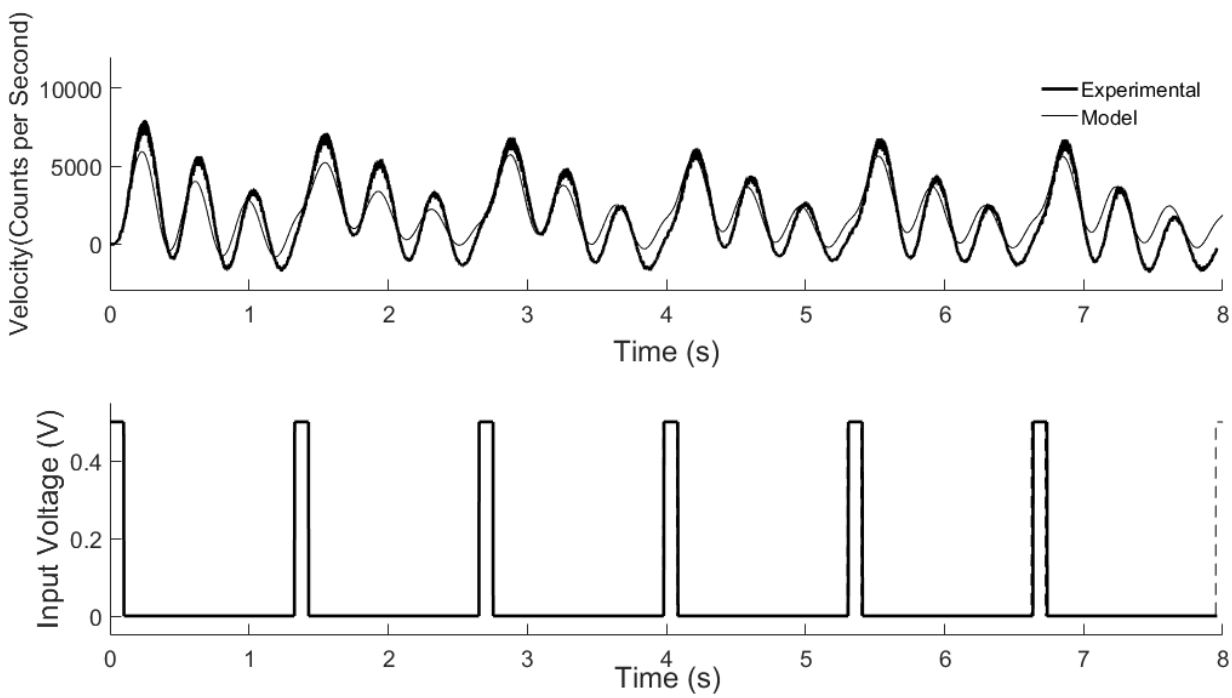

Figure 3: A Comparison of the Angular Velocity Output from Between the System and the Numerical Model with Altered Input. Bottom: The motor received input in voltage with a dwell at zero for 1.225 seconds, and the pulse at 0.5V for 0.1s. The system received torque equal to 0.6 times the voltage. This graph confirms similar inputs between the model and physical system. Top: The output response shows negative feedback resulting from torque application causing destructive interference. The maximum value of the velocity of the first period at $t = 0.25s$ is 8500counts/s, vs. at $t = 6.85s$, v=7083counts/s. One count is 1/16000th of a full rotation. A moving average filter with a smoothing factor of 1/8 reduced the noise in the angular velocity plot for easier comparison.