

Putt Putt Pump

Educational Toy from Thermotopia Enterprises

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Background/Introduction

The concept of the putt-putt boat uses the concept of steam power to create a very simple, two cycle engine which drives a boat through the water. Also referred to as a pop-pop boat, the invention of the simple toy was credited to Thomas Piat [wikipedia], who submitted a patent for the design in 1891. His design differs very little from the current model of a putt-putt boat, consisting of a small boiler and two exhaust pipes, and this simply fabricated boat became a common inclusion in the households of adventurous children and their popularity grew as they became easier to fabricate and distribute. The popularity fell off with the rise of battery powered, plastic toys, however interest in this type of hands on toy has been on the rise lately. This provides a unique business opportunity for Thermotopia to capitalize on the rise of this hands-on concept by returning to a mechanically powered toy.

The Putt-Putt boat utilizes a simple heat engine which powers the boat through steam expansion in the boiler. The engine has two distinct cycles: steam expansion which causes the expulsion of water from the exhaust tubes, and low pressure suction which draws water into the boiler. The energy for this system is provided from the heat energy released from candle wax through burning. The candle is placed under an inboard, metal boiler on the boat and lit. The candle heats the metal boiler to a very high temperature, and at this time any water in the boiler is vaporized upon contact with the hot surface. The initial motion is expulsion, which requires the priming of the boiler with liquid water, and the removal of air from the system. Once the boiler reaches temperature, the water boils almost instantly, and the rapid expansion of the water as it changes to steam pushes the unboiled water in both exhaust tubes outwards with momentum, which propels the boat forwards. This rapid expulsion causes the boiler to become dry and the internal pressure to drop below atmospheric pressure, at which point the favorable pressure gradient between the body of water and the inside of the boiler creates a suction through both exhaust tubes which primes the next cycle of the engine. While a cursory analysis of this process might suggest that the boat should not move due to Newton's third law, however the force imparted on the boat from the water hitting the boiler cancels out the momentum of the reactive force from the water suction. The process works because the net momentum of the outlet

part of the cycle is greater than that of the inlet part of the cycle, which causes a net velocity of the boat forward as the same mass of water is moved in and out of the boiler.

This type of engine is very inefficient, which eliminates it from any practical propulsion system, however it is very impressive to watch and it stimulates curiosity. For this purpose, the boat utilizing this concept became very popular. Another application of this primitive steam engine is as a pump, where the suction and expulsion cycle can be harnessed to draw water from a lower basin and push the water to an upper basin. Watching and understanding this process creates a fun and educational opportunity for kids to learn and experiment, as well as build with their hands and compete with friends.

Problem Definition

2.1 Design Objective

The objective of this design project is to create a pump which harnesses the process of a putt-putt boat engine to pump water against gravity. The projected application of this device is as a fun, safe, and educational toy to teach young, STEM-minded children about practical engineering principles. The most successful pump design will be decided using a head to head competition between the design teams, and that pump will be selected to go to market with.

2.2 Competition Specifications

- Use a steam powered pump to move as much water as possible to the greatest height in a specified time interval.
- Score will be calculated as the product of mass (in grams) and twice the height (in cm); i.e. $\text{score} = 2 * m * h$.
- Each team will have 3 “runs” and the highest score of the three will be the final design score.
- Height will be measured from the top of the water level to the bottom of the receptacle.
- The mass will be measured as the difference in weight between the empty receptacle before the test and the receptacle after the test. Therefore, the receptacle must be able to be detached and measured.

2.3 Competition Arena Specifications

- Water will be pumped from a large basin. With a water height of 5” +/- 1”.
- Water will be pumped into a plastic cup held by a ring. The cup may be modified and the pump may be located anywhere.

2.4 Manufacture and Material Specifications

- The pump must be powered by steam power only.
- No external power sources may be used.
- Total budget for all purchased parts must be less than \$50.

- “Scrounged” materials must include a reasonable price estimate counted towards the total budget.
- Up to 3 cubic inches of rapid prototype material may be used without any effect on the budget, any amount above that will incur a \$15 budget charge per cubic inch.
- Design must pass a design review before manufacturing can commence.

2.5 Energy Source Specifications

- The pump may be powered by a maximum of 3 candle wicks or three oil fueled burners.
- The engine of the pump may be the putt-putt boat boiler supplied with the boat, either modified or not, or it may be a custom built boiler that adheres to the applicable constraints above.

2.6 Geometry Specifications

- The entire design, including all necessary piping, must fit into a 10” by 10” by 10” box prior to testing.

2.7 Setup Specification

- Pump assembly must be completed within 2 minutes (120 seconds) of unboxing.
 - This includes assembly, adjustments, and lighting of candles.
- No adjusting or controlling of the pump are allowed by engineers after the elapsing of the 2 minute set-up window.
- Pumping can begin anytime during the 2 minute setup window, however this will result in early termination of the setup window and the initiation of the pump timer.

2.8 Timing Specification

- The pumps will have 5 minutes (300 seconds) to operate, and at the end of this window, the weight of the cup will be measured to determine the final score.
- This 5 minute window either begins at the end of the 2 minute setup window, or when water starts to enter the receptacle, whichever event happens first.

2.9 Safety Specifications

- The pump must be safe to use and operate.
- The pump may not damage any part of the competition throughout operation.

Design Description

The overall design of the system will consist of a coil heat exchanger, a 3D printed box to hold both the coil and candles, and finally a weighted leg system so the entire pump can be under the water level and fully adjustable with wing nuts. The SolidWorks design can be seen in Figure 1 excluding the leg subsystem. The legs will have slots in them to adjust the level of the pump to maximize its efficiency based on the water level in the system (4"-6" water level). The system needs to reside under the water to increase the water inlet pressure. To do this, the legs will be weighted down to avoid floating and therefore errors in the system and failure in the design. Each specific subsystem design can be seen below in the subsequent sections in the *design description*.

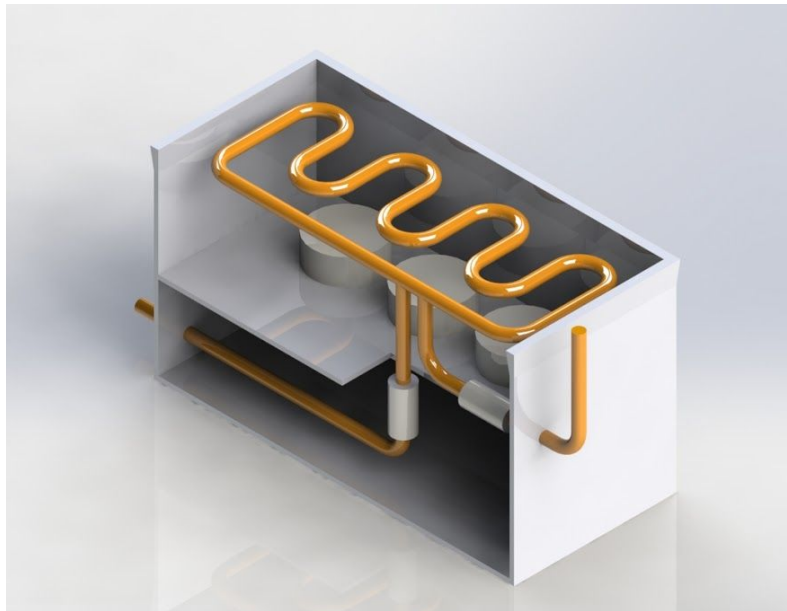


Figure 1: Complete SolidWorks Model.

Coil Design

The coil design was strongly driven by the size of the candles as it was important to promote the highest heat transfer between the candles and the copper coil. The copper coil which was chosen was 3/16" ID, 1/4" OD as we saw this size the most feasible. It was chosen due to the smallest size we could purchase for low price and functionality. The designed coil system can be seen in Figure 2.

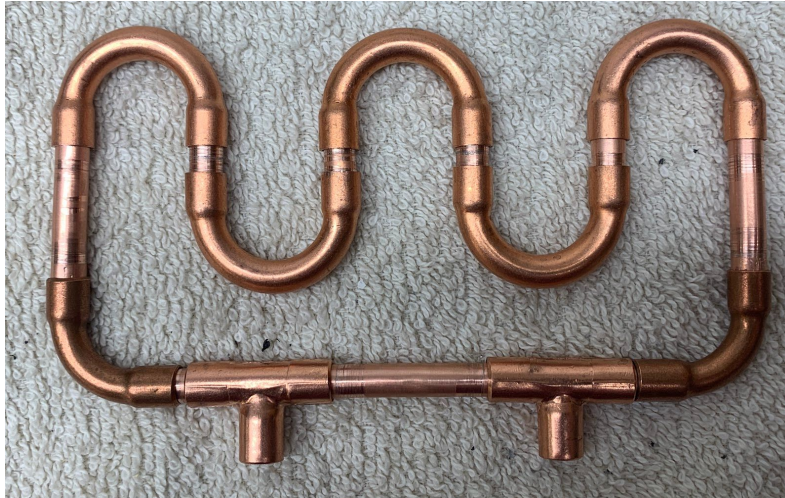


Figure 2: Top coil of heat exchanger system (180° bends are 1” in diameter).

In addition to the copper tubing, a small sheet of copper flashing was going to cover the coil in order to trap more heat and further the heat transfer between the candle and copper heating coil. The thin copper tubing also promoted a smaller amount of water inside in order to heat up a smaller volume. If the coil had too much water inside, it would require too much heat energy to begin the expansion process.

Looking at Figure 1, there are two small white cylinders on the design. These are two check valves to promote directional flow from the inlet to the outlet, not allowing reverse flow. When the water begins to expand, ideally it would flow in both directions, however we forced this flow in one direction.

The first design iteration incorporated a single inlet/outlet design. This design was not effective as it forced water in and out the same hose. To prevent this, a dual inlet/outlet system was created. There will be a minimal mixture of counter flowing fluid to prevent undesired mixing.

System Housing

The boiler, check valves, connection tubes and candles will be housed in a PLA plastic printed container that is submerged in water up to the boiler, with a 0.75 inch lip so that water does not fill the container. The housing acts as a frame for supporting the boiler and tubes, and also a container for the ~2 lbs of weights needed to keep the 53 in³ area submerged under the

water's surface. The printed parts occasionally can leak but the small leaks can easily be plugged with most glues, and so little water is let in that many minutes pass before it is significant.

Incorporated into the system housing will be adjustable legs which will be utilized to change the height of the system based on the 4"-6" variation of water level. This accurate adjustment is crucial as if the system is not at the correct level, it will not function correctly. The leg system design can be seen in Figure 3.

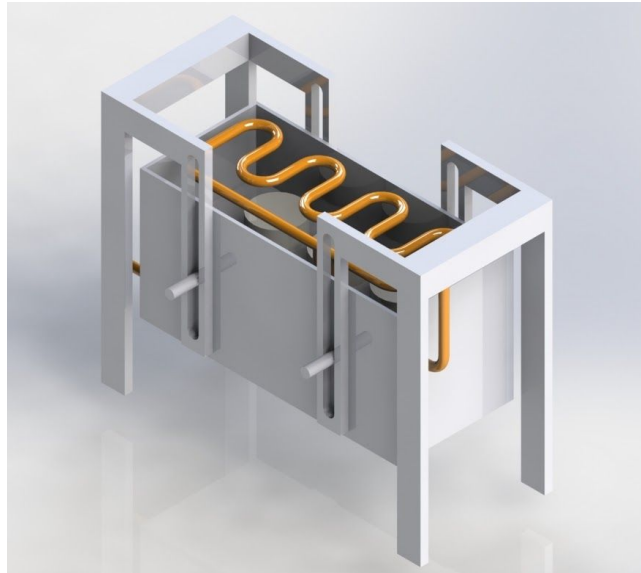


Figure 3: Adjustable leg design for system housing.

The leg system will be a stand-alone unit which connects to the box via the small cylindrical extrusions seen on the front face of the box. The cylindrical extrusions in reality will be threaded rods which will have wingnuts on them for hand adjustment to alter the height of the system housing. If the legs are weighted down enough for full system housing emersion, the height of the system can be moved to wherever the designers wish. The purpose of the wingnuts are to make the system adjustable whenever necessary without the use of tools. It would be changed during the first two minutes of the competition during the preparation period.

The legs are within the 10" x 10" x 10" working frame and do not extend past this working envelope which is given in the design constraints. The leg design is crucial to the entire system as it needs to be relatively precise and adjustable in a quick, effective manner. In addition, the system housing cannot move in the water once the competition has started. The leg

design restricts both of these freedoms to ensure a functioning design and maximum efficiency of the physical system.

Economics

Quantity	Value
Break Even Year	8th Year, 3rd Sector
Maximum R&D Budget	\$3,750,000
10 Year Project Present Value Worth	\$4,265,000
List Price	\$3.12
Quantity Sold Annually	48,000

Table 1: Key Values

Component	Cost	Quantity	Total Cost
<i>1/4 in. Copper Tubing</i>	<i>\$0.015/cm</i>	30	\$0.45
Copper Flashing	$\$0.014/in^2$	6	\$0.08
<i>1/4 in. Vinyl Tubing</i>	<i>\$0.0165/cm</i>	50	\$0.83
Check Valve	<i>\$0.48 ea</i>	2	\$0.94
T-Valve	<i>\$0.10 ea</i>	1	\$0.10
Paraffin Wax	<i>\$0.0054/g</i>	50	\$0.27
<i>1/8 in. Cotton Rope</i>	<i>\$0.0024/cm</i>	3	< \$0.01
Molded Plastic	$\$0.25/in^3$	2	\$0.50
Total Per Unit	-	-	\$3.12

Table 2: System Costs Breakdown

Note: rounding up in the cost section makes some totals too high; total price per unit is adjusted.

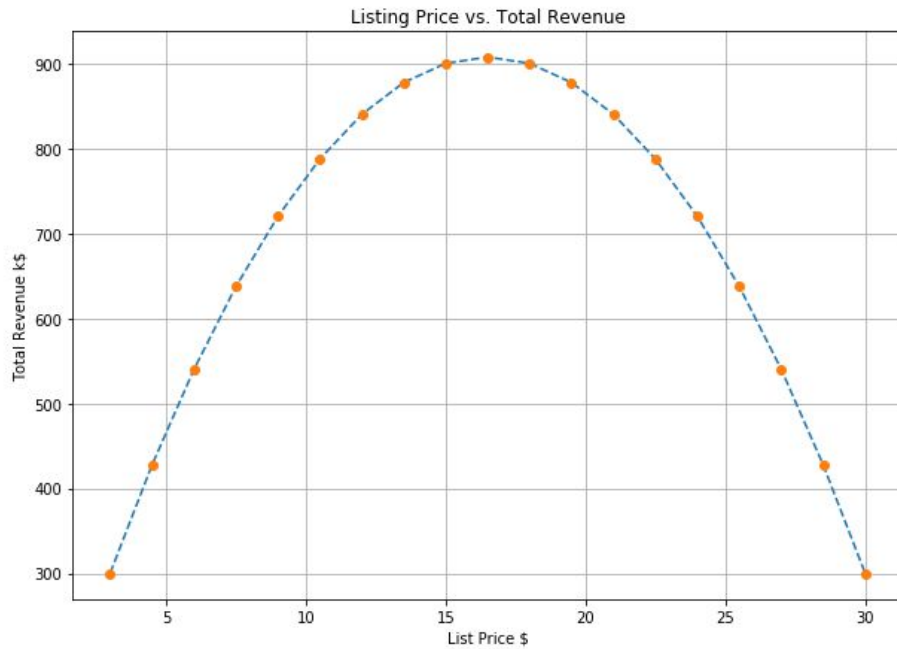


Figure 4: Total Revenue Plot

The highest predicted total revenue falls where we produce and sell 55,000 units per year and charge \$16.50 per unit. When the variable costs are included in this analysis the optimum point is driven to \$18.60 at 3% interest.



Figure 5: Net Yearly Income Plot

Interest rates from 1% to 10% were considered. A working interest rate of 3% was chosen as a reasonable choice for the United States over the next 10 years. The maximum research and development budget can then be derived at 3% interest.

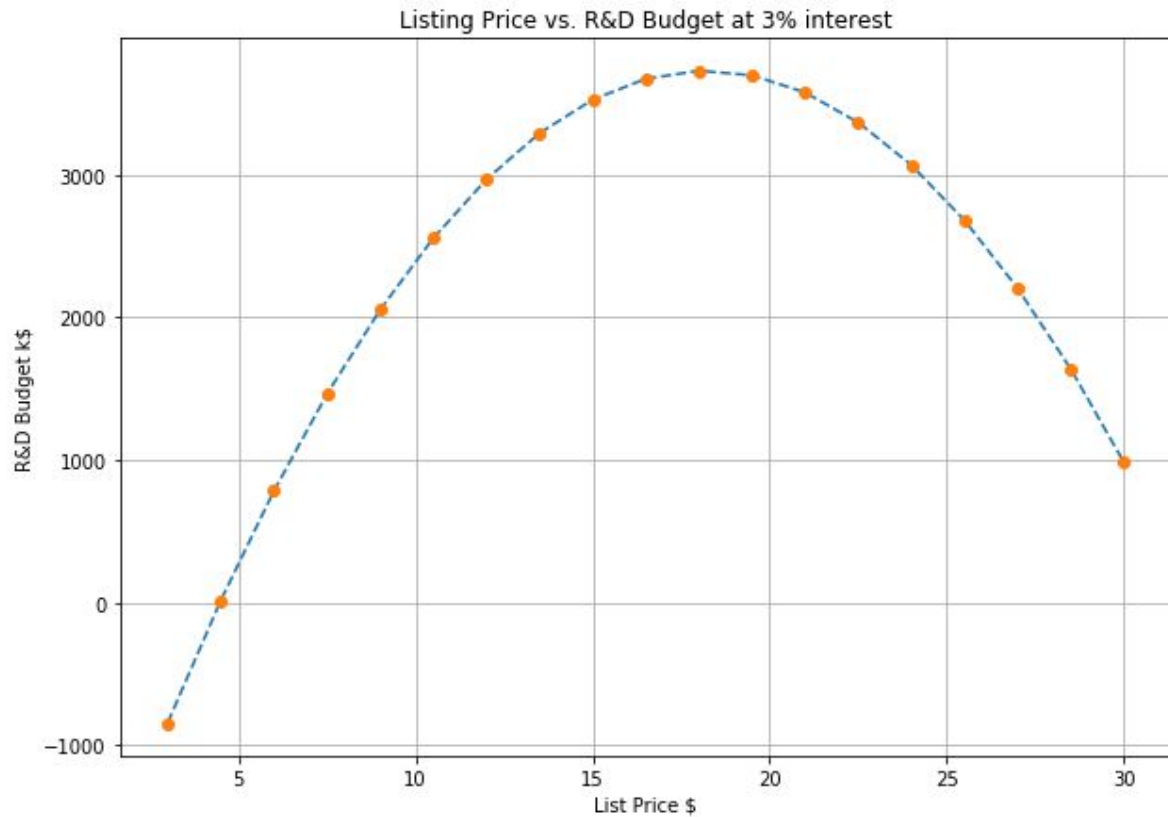


Figure 6: R&D Budget at 3% Interest vs List Price

Of course, the maximum research and development budget falls at the optimal profit point. Ultimately, at a list price of \$18.60 we will sell 48,000 units and have room for as much as \$3,750,000 in research and development spending.

Economic Recommendations

If the interest rate goes up, the ten year present-value-worth of the system drops and the maximum R&D budget will fall as a result. If the economy is experiencing a great deal of uncertainty or is showing signs of recession, cutting this large research and development budget may be a good way to mitigate risks. This would simply increase the maximum rate of return, but provide a bigger cushion if assumptions about the economy fail. Changing the interest rate

should only result in a vertical shifting of the present value worth, so the optimum point would remain the same.

Analysis

Thermal-Fluids

The completion of this design did not include much numerical analysis. Due to the large amount of error associated with this type of power generation, our operating point will be determined mostly through experimental testing.

Our adaptation of the putt-putt concept to a pump application required the system to directionalize the flow, rather than operate as a two-way system as the boat engine does. In order to achieve this goal, two key design parameters were added to the putt-putt design: a T-joint and one-direction check valves.

The T-joint serves the purpose of allowing water to pool without being heated by the flame. This pocket of liquid is ejected from the pump during the flash vaporization of liquid.

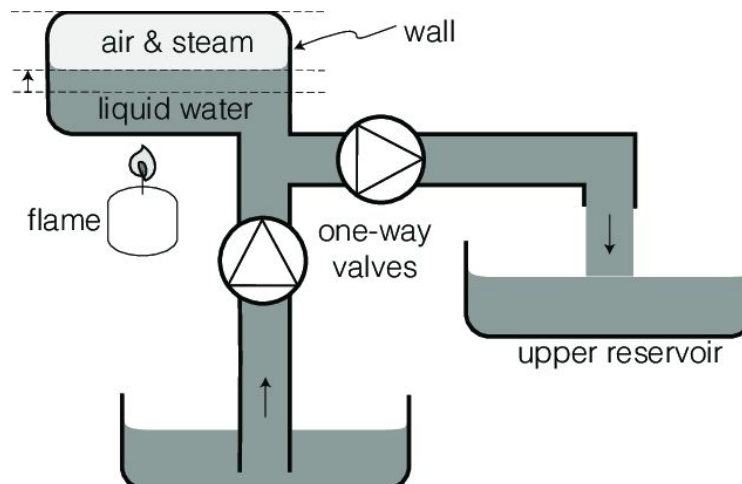


Figure 7: [3] Schematic detailing the operational process of our pump. The lower reservoir is the tank described in section 2.3 and the upper reservoir is the receptacle in section 2.2. The flame is the candle listed in section 2.5.

The check, or one-way, valves serve the purpose of directing the flow from a lower reservoir to an upper reservoir using the power of the expansion of vaporizing water in the boiler, and then allowing water to be drawn in from the lower reservoir to fill the boiler. Assuming that the system is completely filled with water, the force of expansion on the face of the valve plug in the boiler must be enough to overcome the cracking pressure of the system-exit

valve. We assumed that the force resultant from the expansion of the water in the boiler would be enough to open this check valve.

Since the other check valve is located below the surface of the water, the pressure to open the valve can be supplied by the buoyancy force of the water applied on the valve plug face:

$$\begin{aligned} (F_{\text{Buoyancy}})/(A_{\text{Valve}}) &> P_{\text{Cracking}} \\ (\rho_f g V_{\text{Shell,submerged}})/(\pi/4 * (D_{\text{Valve Plug}})^2) &> P_{\text{Cracking}} \end{aligned}$$

If both valves have sufficient pressures supplied to open them, then water will be able to complete the cycle and be moved from the lower reservoir to the upper receptacle without siphoning backwards, thus allowing the putt-putt cycle to be directionalized.

In order to estimate the amount of water that the system can pump to a certain height using the energy supplied from the candle a few parameters need to be measured. First, the mass of candle wax burned in the five minute testing interval, then the efficiency of the boiler can be either measured or estimated. Using literature describing a coil engine that estimates an efficiency of 0.03%, this design applies a Factor of Safety of 10 to reduce the efficiency to 0.003%. Using a simple energy balance:

$$\begin{aligned} \text{Energy}_{\text{Supplied by candle}} &= \text{Energy}_{\text{Required by pump}} \\ \text{LHV}_{\text{Paraffin wax}} * m_{\text{candle burned in 5 minutes}} * \eta_{\text{TH}} &= m_{\text{water moved in 5 minutes}} * g * (h_{\text{receptacle bottom}} + h_{\text{losses}}) \end{aligned}$$

This calculation requires the assumption of a steady average flow. While the engine operates cyclically, with a pulse of flow every cycle. If all the cycles are averaged over the entire time of the test, then that average flow can be treated as a constant process. From this energy balance, an estimated score can be calculated.

Another key component of the system are the weights which prevent free floating of the pump case. The amount of weight required for this task is calculated using the buoyancy force. As discussed before, this system is supported on adjustable height legs which submerge between 80 and 90% of the pump case. The buoyancy force is related to the density of the fluid and the

volume submerged. Including a Factor of Safety of 2, the weight needed on each of the four legs is:

$$W = 2/4 * \rho_f g V_{\text{Shell,submerged}}$$

With a submerged volume of about 55 cubic inches, therefore approximately 11lb of weight per leg is required to hold down the shell.

Economic Analysis

Making good predictions of the profits gained from the development of a toy depends heavily on market research. Because Thermotopia has neither the time nor the resources to conduct such research, sales data from a similar product provided by Thermotopia business partner, Toys”R”us, is used to predict the sales structure.

Price (\$)	3	6	9	12	15	18	21	24	27	30
Quantity (1000 units)	100	90	80	70	60	50	40	30	20	10
Revenue (k\$)	300	540	720	840	900	900	840	720	540	300

Table 3: Sales data

All production values represent the expected amount that can be sold per year. Using these values, we can begin a break-even analysis to find our time to profit, research and development budget, and yearly cash flow. First total revenue is calculated from the unit price and the quantity of products sold.

$$TR = P * Q$$

To get the taxable income (TI) from total revenue (TR), a calculation of the total expenses associated with the manufacturing process is necessary. First the variable costs are calculated.

$$VC = 2.0 * E * Q$$

Because we have chosen to use legal and ethical forms of manufacturing a coefficient of 2.0 is added to the variable costs and the fixed costs are known to be $FC = \$200,000/\text{year}$.

$$TI = TR - VC - FC$$

Accounting for the tax rate, the yearly net income can now be found.

$$I_{net} = TI - TI * \%_{taxes}$$

The net income was maximized using a quadratic regression model from *Desmos* pictured in Appendix B. The equation for revenue, given a cost, was maximized by setting the derivative equal to zero and solving.

From the net yearly income, a present value worth analysis of the project was conducted for a product life of 10 years where the yearly income is treated as a negative annuity. The annuity to present value worth factor is calculated:

$$P/A = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Where i is the interest rate and n is the total number of years.

On this project the research and development budget is based on a desired minimum rate of return of 12%. Therefore we can calculate the maximum research and development budget by simply finding a spending coefficient:

$$R\&D_{coefficient} = (1 - 0.12) = 0.88$$

The maximum research and development budget can be found by simply multiplying the present-value-worth of the project after 10 years by this coefficient.

$$R\&D_{maximum} = R\&D_{coefficient} * I_{net} * P/A$$

Finally the break even year can be solved for by modifying and rearranging the above equation to yield n . In this case, instead of using a coefficient and setting $n = 10$, the maximum R&D budget is known and n is allowed to move. Since n is in the numerator and the denominator as an exponent, an analytical solution, if possible, is not worthwhile. Instead this is solved in the economic design description section graphically using excel.

The Risks of Using Child Labor

On this venture, Thermotopia has the opportunity to outsource our production to Elbonia where labor is extremely inexpensive, but only because of the use of child labor. Despite a clear potential for profit, this opportunity should absolutely not be pursued.

A shallow economic analysis of the situation shows potential for significant gains because the variable costs associated with manufacturing using child labor are only 55% of the full manufacturing cost. However, this economic picture relies on the assumption that the demand curve for Thermotopia's product closely follows the demand curve for a similar product. If Thermotopia's use of child labor becomes known to the public it would be extremely offensive to parents who are buying these toys for their own children. A boycott of our product would both damage the feasibility

Good engineering designs should be carried out *ethically* and *legally*. Going by the spirit of the law, outsourcing this labor to Elbonia is not legal. Section 307 of The Tariff Act of 1930 (19 U.S.C & 1307) explicitly bans the import of goods manufactured using forced or indentured child labor. While it may be possible for Thermotopia to successfully deceive the United States Government or find some loop-hole that allows labor done by foreign children to be considered consensual, doing so would be highly unethical.

As a country, the United States has decided, from experience, that the use of child labor in industrial manufacturing is inherently exploitive. American children under the age of 18 are completely prohibited from taking part in "hazardous occupations" which include mining and manufacturing. These laws exist in order to avoid putting young people in the way of danger they are not yet mentally or physically equipped to deal with. Choosing to outsource this labor to Elbonian children would put them in the way of hazards that children deserve to be free from. All human beings deserve the same basic human rights regardless of their economic status, and intentionally *underpaying* impoverished children to do Thermotopia's manufacturing should absolutely not be under consideration.

Experimental Verification and Proof-of-Concept Tests

Boiler

The boiler in the Put Put Impumpulator comprises bends of 0.25" OD copper tubing fit with check valves that guide the flow from the lower reservoir to the higher reservoir. The design has candles underneath this boiler that vaporize water to raise pressure in a cyclic fashion, where the check valves are below the water level so the high pressure pushes water/gass up out to the higher reservoir before water is sucked in from the lower reservoir. Our testing to make sure this concept does produce a working cycle used one large boiler with one tube leading to a junction. The junction then leads to the lower and higher reservoirs, with check valves guiding flow. The test used tongs to hold the large chamber over small flames for 5 minutes, and concluded that both the ends of the metal pipe did not get hot enough to melt the plastic of the connecting tubes, and that this did produce a cycle, though it was unoptimized and slow.

Future testing will use our design including a boiler that is of smaller diameter, meaning the larger surface area to volume ratio promotes heat transfer. The faster vaporization of water makes the cycles run faster, producing a higher flow rate and pump head. The bottom of this thin tube will be roughed up with sandpaper to increase surface area, again to elevate heat transfer, and copper flashing will be placed above the meandering tube to prolong the boiler's contact with the hot gasses escaping the flame. The tests must be repeatable and have a common order of events, so each test will operate with the following procedure:

1. Check seals
2. Test valves
3. Visual system inspection
4. Test custom candle
5. Prime boiler
6. Insert candle
7. Start pump

Custom Candles

The purchased product used the correct fuel we wanted, paraffin wax, but the wicks did not move a high amount of wax, and could be improved. By replacing the thin unknown wicks with slightly thicker cotton string, the energy output was dramatically increased with a small change. This is due to cotton burning slightly hotter and being great at wicking wax up to the flames. By letting the candles burn for 5 minutes and weighing them before and after, the mass flow rate of wax and therefore an estimated energy output was obtained for configurations of cotton wicked candles with 1, 2, and 3 wicks per candle. The tests were repeated several times for each candle type.

Design Considerations and Constraints

Health and Environmental

The suggested system uses paraffin wax with a cotton wick, copper tubing, plastic check valves, and a plastic PLA printed frame. Producing these in large quantities for profit makes the materials and their production a very important consideration, balancing pollution and health risks with low price materials. These are being marketed as an educational tool towards children, afterall, so a large constraint is being harmful to children. An important consideration is how environmentally friendly we are.

The proposed design uses paraffin wax for its economic viability and high temperature burn, which is better for heat exchange and therefore the power output of the pump. The risks of using paraffin wax concern that the material is produced from petroleum, so its environmental effects don't start at the burning of the candle, but at the drilling and refining of oil as well. As the candle is burned, it releases fumes and soot related to the bot the material wax and wick, the soot from the inefficiency of the burn. Paraffin does release harmful fumes that pose a health risk, but only in such small quantities that working at the factories can cause long term health risks like lung cancer and asthma to long time workers [1]. Burning one several times will not impact the children using it. Additionally, the pump's top plate above the meander of the copper tubes will catch the majority of the soot, as to not have that dissipate in the air.

There are alternatives for paraffin wax, such as beeswax or soy wax, though they both burn at lower temperatures and are more expensive. Another potentially attractive alternative is using liquid fuel. These oils wick up the wicks, reducing waste because almost all the oil is used leaving behind little waste, and they burn slightly more efficiently than solid fuels and don't require thermal energy to melt them before use, like a solid would. The main reason not to use liquid oils is the logistics of containing the liquid with no leaks, and it would be messy and much more difficult to replace.

The other materials are of negligible impact compared to the wax, though most large scale production causes large amounts of pollution. The PLA frame is made from corn byproduct and is even technically edible, harmless while printing and a safe material for use around kids.

The copper tubing and check valves would be bought elsewhere and shaped/fitted in house, requiring little environmental emission except for in transport.

Heat Transfer and Thermodynamic

The best way to increase the efficiency of the system, and therefore its power output if it already works, is aiding heat transfer from the candle flame to the boiler. This can be done by increasing the overall surface area and temperature difference, equating to rising the flame temperature and interaction area, or increasing the boiler's surface area. To prove during testing, the proposed design raises flame temperature by using a similar wick material in the candle that burns hotter, thin cotton string, which also wicked more wax to the flame and increased the flame size. Surface area on the boiler is effectively increased by roughing up the surface and adding a copper plate above the boiler pipe meander.

Social

This product is for the education and interest of children, to introduce them to a steam cycle and to be a fun project. The impact Thermotopia wants to have is to spur development of the next generation of engineers, and a great way of doing that is both introducing it early and using something simple that they can copy. Our simple design leads kids to see that they can replicate this with a little experimentation, which is a fast way to get kids invested. This product would have no negative social ramifications, instead benefiting the teaching of classes and individuals.

Other Considerations

The few other considerations included were economy of scale and ease of manufacturability, both to reduce prices and increase ease of assembly and repair. The materials were chosen because they either have fairly inexpensive analogs when ordering in extreme bulk, or are fairly inexpensive already. Something like the PLA printed housing can be melted down and reused, or replaced fairly easily. No other large scale considerations significantly affected the design.

Conclusions and Recommendations

Overall, this is a simple and effective cycle made from largely safe materials, that is predicted to cost optimally \$3.12 per unit, selling before breaking even on the initial investment of manufacturing infrastructure and shipping. The pump operates similar to a put put boat with check valves, the expansion of water as it is vaporized opening the check valve to push water up, leaving a low pressure in the boiler that opens the other check valve to pull water in, in a cyclic pattern. This configuration based upon pump head calculations and heat transfer estimations can yield a maximum of 45cm in water head at a maximum flow rate of 1.667 g/s. Using the metric of a contrived point system within the design competition, this equates to a maximum score of 46,700 points. We are confident that our device can achieve this score, and that achieving this score will put our product ahead of the other concept designs.

We recommend marketing the Put Put Impumpulator as an educational demonstration and toy to stimulate interest in the next generation of young engineers. Specifically, replaceable components and a simple system promote experimentation and learning, creating the perfect instrument to captivate young minds. A recommended discount for in-class demonstrations like school purchases guarantees this purpose will be fulfilled and a more permanent place in the market.

References

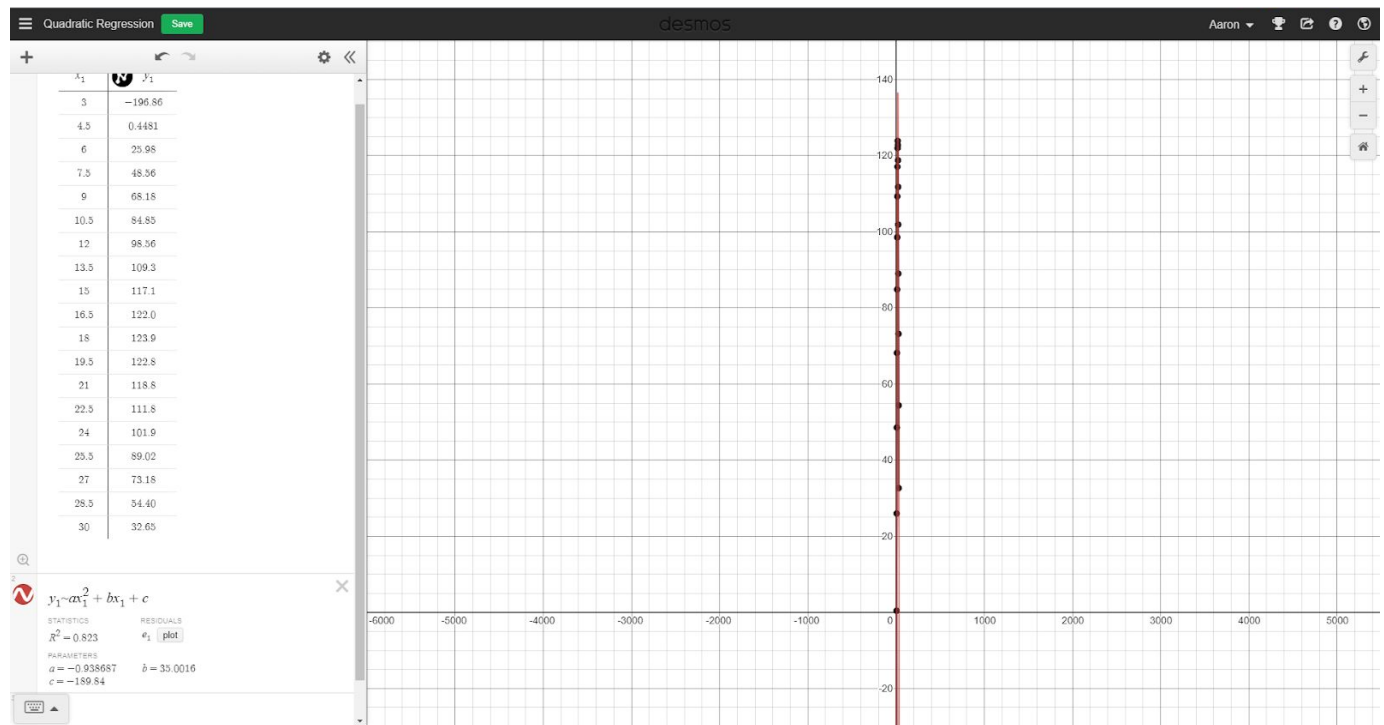
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Appendix:

Appendix A: Break Even Analysis

n (year)	P/A	Annuities	PVW		R&D Costs =	3,753,289
0	0	500,000	0		i =	0.03
0.25	0.2454154618	500,000	122707.7309			
0.5	0.4890240612	500,000	244512.0306			
0.75	0.730839101	500,000	365419.5505			
1	0.9708737864	500,000	485436.8932			
1.25	1.209141225	500,000	604570.6125			
1.5	1.445654428	500,000	722827.2142			
1.75	1.680426312	500,000	840213.1558			
2	1.913469696	500,000	956734.8478			
2.25	2.144797306	500,000	1072398.653			
2.5	2.374421775	500,000	1187210.888			
2.75	2.602355642	500,000	1301177.821			
3	2.828611355	500,000	1414305.677			
3.25	3.053201268	500,000	1526600.634			
3.5	3.276137646	500,000	1638068.823			
3.75	3.497432663	500,000	1748716.331			
4	3.717098403	500,000	1858549.201			
4.25	3.935146862	500,000	1967573.431			
4.5	4.151589947	500,000	2075794.974			
4.75	4.366439478	500,000	2183219.739			
5	4.579707187	500,000	2289853.594			
5.25	4.79140472	500,000	2395702.36			
5.5	5.001543638	500,000	2500771.819			
5.75	5.210135416	500,000	2605067.708			
6	5.417191444	500,000	2708595.722			
6.25	5.62272303	500,000	2811361.515			
6.5	5.826741396	500,000	2913370.698			
6.75	6.029257685	500,000	3014628.843			
7	6.230282955	500,000	3115141.478			
7.25	6.429828184	500,000	3214914.092			
7.5	6.627904268	500,000	3313952.134			
7.75	6.824522024	500,000	3412261.012			
8	7.01969219	500,000	3509846.095			
8.25	7.213425421	500,000	3606712.711			
8.5	7.405732299	500,000	3702866.15			
8.75	7.596623325	500,000	3798311.662			
9	7.786108922	500,000	3893054.461			
9.25	7.974199438	500,000	3987099.719			
9.5	8.160905145	500,000	4080452.572			
9.75	8.346236238	500,000	4173118.119			
10	8.530202837	500,000	4265101.418			

Appendix B: Quadratic Regression Model



Appendix C: Economics Code/Additional Economic Analysis

Attached on following pages